## CS 3120 Quiz Day 4

This packet contains the quizzes for this quiz day. This **cover sheet** is here to provide instructions, and to cover the questions until the quiz begins. **do not open this quiz packet** until your proctor instructs you to do so.

You will have the 1 hour to complete these quizzes. Each quiz is two pages (front and back of one sheet of paper) worth of questions. Make sure to **write your name and computing id at the top of each individual quiz**.

When you are finished, simply submit this packet at the front of the classroom.

This quiz is CLOSED text book, closed-notes, closed-calculator, closed-cell phone, closed-computer, closed-neighbor, etc. Questions are worth different amounts, so be sure to look over all the questions and plan your time accordingly. Please sign the honor pledge below.

*In theory, there is no difference between theory and practice. But, in practice, there is.* 

## THIS COVER SHEET WILL NOT BE SUBMITTED FOR GRADING. DO NOT PUT WORK YOU WANT GRADED ON THIS PAGE

## **Quiz - Module 4: Turing Machines**

## Name

1. [10 points] Answer the following True/False questions.

A <i>Turing Machine</i> contains a finite set of states $Q$ , but an infinitely sized tape	True	False
For language $R$ and string $w$ , a <i>Recognizer</i> will always <i>accept</i> when $w \in R$ but might loop forever when $w \notin R$	True	False
A <i>Turing Machine</i> makes a decision on what transition to follow based on the current state and the symbol currently on the tape at the head position	True	False
NTMs can recognize functions that DTMs cannot	True	False
Multi-tape Turing Machines can recognize more functions than Deterministic Turing Machines if they have $n$ tapes, where $n$ is the size of the input	True	False
Detecting whether the language of two DFAs is equivalent is Decidable	True	False
Suppose I alter <i>TM</i> s so that they can move the head up to <i>five times</i> per step. This <i>TM</i> would be <i>MORE</i> powerful than a standard <i>TM</i>	True	False
Recognizers will always loop forever on NO instances	True	False
If a problem <i>A</i> is <i>decidable</i> , then $\overline{A}$ is <i>recognizable</i>	True	False
If a problem A is <i>recognizable</i> but <i>undecidable</i> , then $\overline{A}$ is <i>unrecognizable</i>	True	False

2. [2 points] Suppose I have two recognizers for languages  $R_1$  and  $R_2$ . Is it possible to construct a *recognizer* for  $R_1 \cap R_2$ ? Briefly explain why or why not.

3. [2 points] Consider the following problem, which is related to *Hilbert's 10th Problem*: Given a polynomial P over n variables  $\{x_1, x_2, ..., x_n\}$ , return true iff there are exist no integer roots for P (i.e., all integer values over  $\{x_1, x_2, ..., x_n\}$  do NOT evaluate P to 0). Is this problem *co-recognizable*? Explain why or why not.

The problems on this page all involve the same problem. Consider the function  $TM_{eq}(M_1, M_2)$ , which returns *True* if and only if  $\mathcal{L}(M_1) = \mathcal{L}(M_2)$ .

4. [6 points] Show that  $TM_{eq}$  is undecidable by providing a reduction from  $A_{TM}(M, w)$ . We will split this reduction into two parts:

First, given input to  $A_{TM}$  (i.e., a machine M and string w), describe (write the pseudocode for) how to construct two machines  $M_1$  and  $M_2$  (one machine per box). You should do this such that  $\mathcal{L}(M_1) = \mathcal{L}(M_2)$  if and only if Maccepts w. HINT: One of your two machines will be trivial (it will either always reject or always accept, the other is a function of M and w)

Now, assuming the construction above works, describe the pseudocode steps for a decider to the *undecidable* problem  $A_{TM}$ . You may invoke your construction from the previous problem by simply stating *Construct*  $M_1$  and  $M_2$ . Assume the construction works even if your answer to the previous problem isn't correct.

We provide the beginning of the proof here, which leads into the description of your decider: Assume that  $TM_{eq}$  is decidable, thus a decider  $M_{eq}$  exists that decides it. We can construct a working decider for  $A_{tm}$  as follows: